Comparison of the low-frequency variations of the vertical and horizontal components of the electric background field at the sea bottom

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ABSTRACT

Natural electric field variations are measured at the sea bottom over long periods of time by means of stationary, vertical, and horizontal galvanic antennas. We compare the power spectra of the vertical and horizontal field components and the extent to which they may be reduced by standard averaging techniques. Although the raw spectra of the vertical and horizontal components do not differ greatly, the difference in the spectra after averaging is significantly greater. Most significantly, in the frequency range between 0.0005 and 0.03 Hz, this averaging scheme suppresses the vertical electric field component more strongly than the horizontal component.

INTRODUCTION

The electromagnetic signal that reaches the bottom of the sea is of fundamental interest in a range of contexts. Although the field may have an intrinsic interest, to the extent that it reveals the nature of its sources, it is perhaps more commonly measured for the purpose of revealing the geologic structure that affects it. In the case of magnetotellurics, the natural electromagnetic field variations are used to detect resistive or conductive anomalies in the subsurface. This is also the objective in the case of marine controlled-source electromagnetic measurements (CSEM) where a transmitter generates the controlled field. The subsequent measurements rely on the resolution of a signal which must be stronger than the natural background field. To achieve this, various averaging techniques are applied to reduce the noise and enhance the signal to noise ratio. However, not all frequencies of the noise may be removed. For this reason, the noise in some ranges of frequency is efficiently removed, whereas the noise in other frequency ranges survives. Our purpose in this paper is to quantify and compare the effect of such averaging procedures on the vertical and horizontal components of the sea bottom electric field.

Electromagnetic noise has a wide variety of generating mechanisms. Magnetotelluric noise comes from electromagnetic activity in the ionosphere as well as lightning around the earth. However, the noise that reaches the sea bottom may be due to a large number of mechanisms apart from this. E-field variations due to ocean surface waves have been studied by many authors, including Longuett-Higgins (1950), Weaver (1965), Sanford (1971), Podney (1975), Cox et al. (1978), Chave (1984), Davey and Barwes (1985), and Ochadlik (1989), and their effect typically enters below the frequency of 0.1 Hz. In fact, any frequency component of an electromagnetic signal with frequency that comes from the sea surface, or sources above it, will be damped as $e^{-z/\lambda}$ where $z$ is the depth and the skin depth $\lambda = \sqrt{\rho/(\pi f \mu_0)}$. Here, $\rho$ is the sea water resistivity and $\mu_0$ the magnetic permeability of vacuum. This implies that, at any given depth, the field with frequencies above a certain threshold frequency must have its origin in sources below the sea surface, or in the measurement apparatus itself.

Long-term measurements of the electromagnetic effect of ocean currents have been carried out by Liley et al. (2004), and the electromagnetic effect of internal waves have been analyzed by Petersen and Poehls (1982) among others. A theory for the electromagnetic effect of microseisics was put forward by Longuett-Higgins (1950) 60 years ago, and the effect of Rayleigh-Stoneley waves has been studied by Webb and Cox (1982). Recently, Håland et al. (2012) demonstrated that ocean surface waves give rise to noise mainly in the $E_z$-component on the sea floor, whereas other sources appear to dominate in the horizontal field components.

Only recently has it been possible to measure the relatively weak vertical electric field component with any accuracy, because even minute deviations from the vertical antenna orientation will cause contamination of the signal. Our measurements are accurate in resolving field strengths down to 0.5 nV/m, and in terms of verticality of the receiver antenna, as it is aligned with the direction.
of gravity to within 0.1°. This allows for a separation between horizontal and truly vertical field components, which, to our knowledge, is unprecedented.

Generally, the removal of noise is easier outside the frequency range of the transmitted signal (e.g., Zhdanov and Keller, 1994). In frequency domain techniques, where the signal is transmitted at some fixed frequencies, the frequency range is centered around the few frequencies that are transmitted. In time domain techniques, where measurements are done over a certain listening time, typically 2–8 s, in a silent transient period after the transmitter is shut off, the troublesome frequency range is around the inverse listening time. It is therefore of fundamental and technological interest to quantify the effect of averaging techniques on the background noise.

There are many ways of reducing unwanted noise in an EM signal (e.g., Zhdanov and Keller, 1994). For instance, the long-range correlation in the horizontal directions of the magnetotelluric signal makes it possible to reduce noise by subtracting the signals at different receiver positions. In this paper, however, we focus on one point measurements of the electromagnetic background field, with particular emphasis on the relatively weak vertical component, which, to our knowledge, has not been recorded with comparable accuracy previously. To illustrate the effect of how noise may be reduced by targeting certain correlation times, we perform a stacking procedure of the data, and compare the results for the different field components. The main conclusion of the paper is that, in the frequency range between 0.0005 and 0.03 Hz, this averaging scheme suppresses the vertical electric field component more strongly than the horizontal component.

In the rest of the paper, we first carry out an analysis of only the vertical component and introduce the averaging schemes. This analysis shows that binning efficiently removes noise only up to the point where the binning time approaches the smallest relevant correlation time. Then, a comparison based on power spectra is done between the vertical and horizontal components. This comparison demonstrates that, although the ratio of the power spectra is fairly close to unity at high frequencies $f > 0.05$ Hz, the variance of the horizontal component is larger than the corresponding quantities of the vertical component by more than an order of magnitude.

MEASUREMENTS AND NOISE REMOVAL

The sea bottom measurements are obtained by the recently developed equipment of PetroMarker. This equipment is designed mainly for the challenging measurements of the vertical field component, which requires antennas that are carefully screened and vertical to within 0.1°.

The present data is acquired by means of 3- and 10-m-high tripods positioned on the sea bottom from a ship at water depths of $d = 106$ to $122$ m and $d = 313$ m (see Holten et al., 2009). The vertical tripod antenna is suspended like a pendulum with lead-chloride electrodes at the extreme of this pendulum arm. This is shown in the leftmost Figure 1. The rightmost Figure 1 shows the screened pendulum with its fabric coating as it is ready to be placed in the sea. The screening shields the antenna from hydrodynamic forces, and is a key factor in ensuring verticality and in reducing hydrodynamic noise. The horizontal antennas are located in the base of the structure and the three sets of horizontal electrodes are seen as white cylinders on the base poles. The electrodes as well as the electronics, such as the analog-digital converter are identical for the horizontal and vertical measurements. The electrode separation of the horizontal receivers is 4.9 m, or half the separation of the vertical receivers. This translates to a factor of two between the internal noise level of the horizontal and vertical electric field components. However, it is observed that only in the high-frequency range may the internal noise dominate and the contribution to the signal variance comes from the low-frequency range. The instrument noise in the voltage includes the basic, unavoidable Nyquist noise, and has a roughly white spectrum with an amplitude of 10 nV/$\sqrt{\text{Hz}}$ in the frequency range above 10 Hz. Below that, the instrument noise includes electrode drift at the very low frequencies and the instrument noise has a typical $1/f^\alpha$ behavior. Under favorable sea conditions, the low-frequency instrument noise may be observed in the vertical component measurements. In all measurements, the data are low-pass filtered to frequencies below 350 Hz. The vertical and horizontal E-field components are measured over a period of 40 hours and the power spectra are calculated.

Figure 2 shows a characteristic measurement of the vertical field component taken while there is no transmission; only the natural background noise. The data sampling rate is a 1000 Hz. The noise has a wide range of frequencies because there is the rapid electronic noise, some oscillations with a period of order 18 s, most likely caused by ocean waves (see Håland et al., 2012), and an overall drift in the data of roughly 100 nV, which is usually associated with electrode drift. The noise causes the field values to drift over more than 100 nV/m in three minutes, but we require accuracies around 1 nV/m.

In the CSEM context, noise processing is done by averaging. Although these averaging techniques are motivated by time-domain, or transient, pulsing systems, like those employed in the PetroMarker system, they are also well-suited to remove high- and low-frequency components in any measurements carried out over time at a stationary position. The high-frequency noise is reduced by binning, i.e., averaging in specified time-windows. The low-frequency noise is

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**Figure 1.** (a) Sketch of the 3-m tripod antenna structure and (b) the 10-m tripod. Both antennas are covered by screening fabric.
reduced by stacking, which is a process that averages measurements that are separated by longer times. Each such stack will eventually be averaged itself, thus reducing the low-frequency noise further. This averaging procedure is a direct average of identical and subsequent pulse sequences carried out at the same location.

The stacking procedure is created to remove systematic drift in the data. For this purpose, the transmitter signal changes sign so that different signals may be subtracted, rather than added, during averaging. Figure 3 shows the sampling functions used in this procedure. This sampling function is just a graphical representation of the so-called Thue-Morse sequence. The $p_2(t)$ sampling function, for instance, defines a P2 sequence. It samples for a period $T_2$, and this corresponds to the listening time after the first transmitter signal. Then, after a larger time $T_2$, the response to the inverted signal is recorded and subtracted from the first. If $f(t)$ is an arbitrary signal, then the P2 measurement is defined as

$$f_2(t) = p_2(0)f(t) + p_2(t + T_2)f(t + T_2)$$

$$= \frac{1}{2}(f(t) - f(t + T_2)).$$

Likewise, the result of a P4 sequence is

$$f_4(t) = \frac{1}{2}(f_2(t) - f_2(t + T_4)),$$

and so on. Now, if $f_i(t)$ is written as a Taylor series

$$f_i(t) = \sum_{n=0}^{\infty} a_i^{(n)}(t - t_i)^n$$

it is easily seen that the Taylor series of $f_{i+1}(t)$ is

$$f_{i+1}(t) = \sum_{n=0}^{\infty} \frac{1}{2} [a_i^{(n)}(t) - a_i^{(n)}(t + T_{i+1})]n,$$

where the $a_i^{(0)}$-term cancels, and the constant term is proportional to $a_i^{(1)}$. This means that every time $i$ is increased by one, the constant coefficient goes away and the coefficient of the linear term becomes the constant coefficient. This means that if the signal has the form

$$f(t) = a_i^{(0)} + a_i^{(1)}t + a_i^{(2)}t^2 + g(t),$$

with $g(t) = o(t^2)$, the constant term will be eliminated in $f_2(t)$, the linear term in $f_3(t)$, and the quadratic term will be eliminated in $f_4(t)$. In other words, if $g(t)$ is a rapidly varying function for which the stacking is essentially just an averaging procedure, the stacking eliminates the drift terms up to increasing order in time as the stacking order increases. Correspondingly, when we replace the above $f(t) \to E(t)$, then $E_2(t)$ will not carry the linear drift in the electrode signals, or even the quadratic in time drift that would come from a linear change in the drift rate. Figure 4 shows how P8 stacking reduces the drift in the data. The black dots show the result of stacking alone, and now the typical spread in the data is reduced to 10 nV. This spread is reduced further by averaging the data into bins. When the bin size is increased from 100 to 1000, the spread is not significantly decreased because now some slower oscillations dominate the rms-spread in the signal. The black line shows the P8 stacking and binning of the signal from a 10-m tripod, and it is seen that the variations are somewhat smaller in this case. The 3- and 10-m tripods are very close, and the interesting observation in the figure is that the low-frequency oscillations appear to be in phase, indicating that the oscillations are not linked to the tripods themselves but the external fields. The frequency is consistent with that of surface sea waves with a wavelength of roughly 500 m (Håland et al., 2012). The binning and the P8 averaging are carried out as so-called running averages. More specifically, the binning around a time $t$ is defined as

$$\bar{E}_B(t) = \frac{1}{N_B} \sum_{i=-N_B/2}^{N_B/2} E(t + i\delta t),$$

where $N_B$ is the number of measurements per bin and $\delta t$ is the time-step, which is normally 1 ms. Similarly, the P8 stacking may be done for every $t$ so that there is no loss in the number of $i$-values in these averaging steps. Note, additionally, that the binning and stacking procedures are linear, so that the order in

![Figure 2. Raw data: The vertical electric field component $E_z$ measured by a 3-m tripod at 300-m water depth during a routine test. The red line shows a linear fit to the data.](image)

![Figure 3. The sampling functions defining the different P2, P4, and P8 sequences.](image)
which they are applied is immaterial. The shift times $T_8 = 2T_4 = 4T_2 = 8T_1$ where $T_1 = 3$ s, so that the entire P8 sequence is over in 48 s. When the running average is applied, one must make sure that there are at least 48 seconds of additional data beyond the last point of averaging.

In the last averaging step, however, there is a reduction of the number of data points by a factor of $N_g$, where $N_g$ is the number of P8 sequences needed to cover the data set. This is seen in Figure 5, where time only runs over slightly less than a minute, and where a P4-averaged signal is added. In the P8 case, the average is over three sequences, whereas the P4 averaging correspondingly allows for an average over six sequences. There is no striking qualitative difference between the P8 and P4 graphs. Apart from the somewhat more erratic nature of the P4 curve, the variance, which we eventually shall use as an estimate of the error bar, is almost the same for the two curves. The error bar in measured $E_z$-signals may be estimated as the rms deviations in the signal after all the averaging steps are carried out. This may be computed from the signals shown in Figure 5. In Figure 6, the rms-variations in the stacked/binned/stack-averaged data of Figure 5 and the stacked/binned data of Figure 4 are shown. The reduction in noise due to the averaging over the three subsequent stack sequences is consistent with the $1/\sqrt{3}$-factor that would result from independent, random measurements. On the other hand, the decay of the error bar with $N_g$ is slower than what would result from uncorrelated noise. In the case of uncorrelated (white) noise, the decay would go as $1/\sqrt{N_g}$. However, it is clear from Figure 4 that there is a correlation time given by the quasiperiodic oscillations. As a result, little improvement of the error bar is achieved by increasing $N_g$ beyond 100.

**POWER SPECTRUM IN THE LOW-FREQUENCY DOMAIN**

Magnetotelluric noise appears largely in the horizontal $E$-field components. This follows directly from Maxwell’s equations and the large scale-separation between the thickness and the horizontal dimensions of the atmosphere. This may be deduced from the following well-known argument. Neglecting displacement currents below the sea surface, where they only contribute in the very high-frequency part of the signal, the combination of Ohm’s law and Maxwell’s equations imply that $E = \rho \nabla \times H$, where $\rho$ is the resistivity and $E$ and $H$ are the electric and magnetic field. This means that the vertical field component $E_z = \rho (\partial H_x/\partial x - \partial H_y/\partial y)$ and a horizontal component takes the form $E_x = \rho (\partial H_y/\partial y - \partial H_z/\partial z)$. Because, on the scale of the earth, the ionosphere is essentially a very thin layer, the derivatives are larger in the vertical than in the horizontal directions and $\partial H_x/\partial x \gg \partial H_y/\partial y$ or $\partial H_z/\partial z$. For this reason, $E_z \ll E_x$ or $E_y$. Figure 7 shows the power spectra of the vertical and horizontal field components. Although both spectra may be fitted by a $1/f^2$ function, the vertical spectrum comes much

![Figure 4](image_url)  
Figure 4. The vertical electric field component $E_z$ after P8 stacking (black dots) and stacking and binning (colored graphs). The size of the binning windows are 10 (red), 100 (green), and 1000 (blue) measurement points. The data are the same as in Figure 2 (3-m tripod) except for the black line which shows data from a nearby 10-m tripod, stacked and binned in bins of 1000 points.

![Figure 5](image_url)  
Figure 5. The black line shows the same 10-m tripod data as in Figure 4, but averaged over three consecutive P8 stacks. The red line shows the result of averaging the same data in six consecutive P4 stacks.

![Figure 6](image_url)  
Figure 6. The $E_z$-error bars for the 3-m tripod (red) and a 10-m tripod (black) as a function of the size of the binning window $N_b$. Open symbols show the P8-stacked and binned results, whereas filled symbols show the results of binning, stacking, and averaging over three stacks. The line shows a slope of $-1/2$. 

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closer to such a fit than the horizontal spectrum. Figure 8 shows that the magnitude of the ratio $|\hat{E}_c(f)/\hat{E}_c(f)|^2$ of the two spectra is not very far from unity. However, after P8 averaging, the separation becomes much larger. When it is employed together with binning, it leaves only the noise in the intermediate frequencies. In other words, only the correlation times that are of the order of the duration of the whole P8 sequence are removed.

To understand Figure 9 we need to identify the Fourier filter function that corresponds to the stacking procedure given in equation 1. First, when Fourier transformed equation 1 gives

$$\hat{f}_2(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dt e^{-i\omega t} f_2(t) = \frac{1}{2}(1 - e^{i\omega T}) \hat{f}(\omega). \quad (7)$$

Because $(1 - e^{i\omega T})/2 = e^{i\omega T/2} \sin(\omega T/2)$, we can make the same calculations for the Fourier transforms of equation 2 to get $f_4$ and $f_5$, and this eventually gives the power spectrum

$$|\hat{f}_8(\omega)|^2 = \sin^2(\omega T_8/2) \sin^2(\omega T_4/2) \sin^2(\omega T_2/2) \hat{f}(\omega)^2. \quad (8)$$

or, if we set $T_2 = T$, $T_4 = 2T$ and $T_8 = 4T$, as is done in the actual transmitter pulses, we get

$$|\hat{f}_8(\omega)|^2 = \sin^2(\omega 2T) \sin^2(\omega T) \sin^2(\omega T/2) \hat{f}(\omega)^2. \quad (9)$$

If this formula is applied to the electric field, we immediately get

$$|\mathbf{E}_8(\omega)|^2 = \sin^2(2\omega T) \sin^2(\omega T) \sin^2(\omega T/2) |\mathbf{E}(\omega)|^2. \quad (10)$$

where $|\mathbf{E}(\omega)|^2$ is the spectrum of the measured vertical or horizontal field components shown in Figure 7. Some care is needed in obtaining the theoretical curve in Figure 9, most notably the removal of the linear drift in the data. If the data $f(t)$ is sampled over a time window from $t = 0$ to $t = \Theta$, then this drift removal amounts to the transformation $f(t) \rightarrow f(t) - (f(\Theta) - f(0))t/\Theta$ before the Fourier transform is taken. This procedure ensures we may ignore the boundary terms that come from the fact that in equation 7 the time integration is actually only over a finite domain. In Figure 9, we see a general agreement between this theory and measurements. The red and blue power spectra in Figure 9 are calculated by taking the Fourier transforms after the P8 averaging. Figure 10 shows how the standard deviation varies with the length $T$ of the individual (P1) parts of the P8 sequence. The index 1 on $\sigma_1$ refers to the fact that only one P8 sequence is involved and that no averages over subsequent sequences are taken.

For a random walk signal (which has a $1/f^2$ spectrum), one would expect $\sigma_1 \propto T^{1/2}$ behavior, which is indeed what the $z$-component $\sigma_z$ shows. The $x$-component $\sigma_x$, on the other hand, shows a behavior with a more complex kind of correlation. Figure 11 shows the variance $\sigma^2$ as a function of the number $N$ of P8 sequences used to compute it for the vertical (Figure 11, left figure) and horizontal (Figure 11, right figure) field components. The $1/N$ behavior would
The standard deviation $\sigma$ for the vertical (blue curve) and horizontal (green curve) field components, measured as a function of the period $T$ of the P8 sequence.

The variance $\sigma^2$ as a function of the number $N$ of P8 sequences used to compute it for the (a) vertical and (b) horizontal field components. The $1/N$ behavior is shown by the stapled line.

result if the electric field values were entirely independent from one P8 sequence to the next.

CONCLUSION

We have analyzed the sea bottom electric background field and compared the horizontal and vertical field components, in terms of their power spectra, and in terms of their variance. The variance, which we take to be the error bar in our measurements, is about an order of magnitude smaller for the vertical field component than the horizontal one, in spite of the fact that it is the vertical component that mainly shows an effect of ocean surface waves.

The removal of high-frequency noise is done by standard binning, i.e., a running average carried out over $N_B$ subsequent data-points. As expected, the variance behaves as that of an uncorrelated signal up to the point where the averaging times approach the shortest correlation times. We found that the effect of the P8 averaging procedure on the power spectrum may be represented by a simple multiplicative function as long as the signal is preprocessed by removing the linear drift in it. Although this filter function is just a direct consequence of the Fourier transform procedure, it may be useful in optimizing the noise removal.

As a general conclusion of our measurements, we have observed that the electromagnetic noise is significantly larger in the horizontal, than in the vertical $E$-field components. Moreover, the type of correlations that exist in the vertical and horizontal spectra causes the P8 averaging procedure, which is designed to remove low-frequency noise, to be more effective for the vertical, than the horizontal components. The vertical field component has a cleaner $1/f^2$ spectrum at low frequencies, which is a signature of the same (lack of) correlations that one finds in a classical random walk, or in normal, diffusive Brownian motion.

REFERENCES


